Radiative Processes Final Project: **Bremsstrahlung**

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Abstract

Bremsstrahlung is the radiation produced through ions and electrons undergoing free-free interactions, resulting in a deceleration and subsequent energy loss of the electron. It is an important source of radiation in several areas of astronomy, both relativistic and non-relativistic. Bremsstrahlung is observed as radio waves in HII regions and as X-ray emission from low mass X-ray binaries (LMXB) and the intra-cluster medium (ICM)of distant galaxy clusters. In this report, we present an overview derivation of the physics underlying these processes, and include a description of relativistic corrections and the applications of bremsstrahlung in astrophysics.

Introduction & Underlying Physics

Deriving from the German *bremsen*, "to break" and *strahlung*, "radiation," *bremsstrahlung* is known as "braking radiation." This particular variety of radiation originates from a charged particle decelerating, "braking" when passing near another particle of the opposite charge.

We consider this process from the perspective of an influx of rapidly moving electrons past stationary ions, giving rise to a spectrum of radiation. Although bremsstrahlung from the opposite situation involving decelerating ions is possible in theory, due to the assumptions described below, we ignore these considerations for the scope of this report.

In this section, we first briefly describe the derivation of the processes by which charged particles emit radiation. For the sake of brevity, we provide these principles in an overview fashion so that we do not stray too far from the main topic of bremsstrahlung radiation. We then describe the results of this analysis in the thermal case, assuming a standard Maxwellian distribution of particle velocities.

Approximation charged particles in the normal manner as dipoles, the dipole moment of a given particle is given by $\mathbf{d}(t) = \sum q_i \mathbf{r}_i(t)$. In frequency space, this dipole moment can be expressed through the Fourier transform such that

$$\hat{\mathbf{d}}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{d}(t)e^{i\omega t}\mathrm{dt}$$

which is then extended to an accelerating scenario

$$\hat{\mathbf{d}}(\omega) = \frac{1}{2\pi\omega^2}\int_{-\infty}^{\infty} \ddot{\mathbf{d}}(t)e^{i\omega t}\mathrm{dt}$$

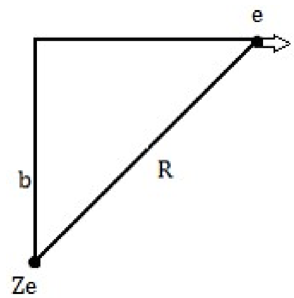
since the dipole moment is sinusoidal. This is given by $\ddot{\mathbf{d}} = \sum q\ddot{\mathbf{r}}_i(t) = \sum q\dot{\mathbf{v}}_i(t)$ Furthermore, knowing that

$$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3}|\hat{\mathbf{d}}(\omega)|^2$$

we derive an expression for power of bremsstrahlung radiation.

We interpret this classically and utilize the necessary correction factors for quantum properties. This will appropriately describe bremsstrahlung radiation.

Classical Bremsstrahlung Derivation



The Figure above is a simple schematic of the situation in which bremsstrahlung radiation arises. An electron moves with a velocity vector past an ion of charge Ze, where Z denotes the atomic number. The particles are a given distance R apart, and are separated by an impact parameter b.

The dipole moment in said scenario would be given by $\mathbf{d} = e\mathbf{R}$, and it follows that $\ddot{\mathbf{d}} = -e\ddot{\mathbf{R}} = -e\dot{\mathbf{v}}$. The Fourier transform described in the previous section will therefore become:

$$\hat{\mathbf{d}}(\omega) = \frac{e}{2\pi\omega^2}\int_{-\infty}^{\infty} \dot{\mathbf{v}} e^{i\omega t} \mathrm{d}t$$

To approximate, we suppose the electron moves very rapidly past the ion, so that angular deviation from a straight path is minimal. The two particles' interaction would occur in a time called the collision time, which is given as the impact parameter divided by the electron's velocity ($\tau = b/v$). The small angle approximation allows us to use the the asymptotic limits of the dipole moment, as opposed to performing the full integration. It is then seen that:

$$\lim_{\omega\tau \ll 1} \hat{\mathbf{d}}(\omega) = \frac{e}{2\pi\omega^2}\Delta\mathbf{v} \quad \text{and} \quad \lim_{\omega\tau \gg 1} \hat{\mathbf{d}}(\omega) = 0$$

Since $\Delta\mathbf{v} = \int_{-\infty}^{\infty} \dot{\mathbf{v}} \mathrm{d}t$, considering exclusively the acceleration component caused by the Coulombic force along the normal line from the electron's motion, we then obtain the following expression:

$$\Delta v = \frac{Ze^2}{m}\int_{-\infty}^{\infty} \frac{b}{(b^2 + v^2t^2)^{3/2}} dt = \frac{2Ze^2}{mbv}$$

we then determine the power that is radiated per unit frequency in the regime in which $b \ll v/\omega$ must be in fact

$$\frac{dW}{d\omega} = \frac{8Z^2e^6}{3\pi c^3 m^2 v^2 b^2}$$

and approximately equal to zero in any other regime.

The scenario described above, however, is only for interactions between one ion and one electron. To develop this analysis for a given volume consisting of n_e electrons in a unit volume and n_i ions in a unit volume, we can similar express:

$$\frac{dW}{d\omega dV dt} = n_e n_i 2\pi v \int_{b_{min}}^{b_{max}} \frac{dW}{d\omega} b \, \mathrm{d}b$$

for a given volume where the differential element of volume is given by $2\pi b \, \mathrm{d}b$ for an electron flux $n_e v$ on the stationary ions.

The upper and lower limit of this integral can then be given such that:

- $b_{\max}$ is defined as where this integral will reach its low asymptotic limit of zero. An approximate but useful limit would be defined as:

$$b_{\max} = v/\omega$$

- The lower limit is defined as where the small angle approximation is no longer valid. This would occur at

$$b_{\min} = \frac{4Ze^2}{\pi m v^2}$$

There exist additional quantum considerations for this effective minimum limit but those will be included below for simplicity and mathematical ease.

Lastly, we acquire the final result of the classic single velocity approximation of bremsstrahlung radiation that gives power radiated per unit frequency per unit volume per unit time, given by:

$$\frac{dW}{d\omega dV dt} = \frac{16\pi e^6}{3\sqrt{3}c^3 m^2 v} n_e n_i Z^2 g_{ff}$$

where g_{ff} is known as the Gaunt factor. The Gaunt factor a corrects for additional quantum effects and is determined by the frequency and energy in the specific scenario. The main consideration to note in this proceeding is that radiated power (denoted below as P) can be given by:

$$P \propto \frac{n_e n_i Z^2}{m^2 v}$$

And given this it follows that while bremsstrahlung caused by the deceleration of an ion would indeed be possible, it would need on the order of 10^6 times more power as the radiated power is proportional to $1/m^2$ and the proton is 2000 times more massive than the electron. Therefore, it is reasonable to neglect these scenarios.

Thermal correction

Thus far only the case in which electron flux is moving at the same uniform speed past ions has been considered. In practice, electrons will have a distribution of velocities. It is conventional to assume that the spread in velocity will be given by a Maxwellian distribution, which varies with temperature.

We therefore conclude that the probability of finding a particle between the velocities v and $v + \mathrm{d}v$ must be proportional to the following functional form:

$$v^2 \exp\left(\frac{-mv^2}{k_B T}\right) \mathrm{d}v$$

Using this fact, we extend our analysis of bremsstrahlung radiation using this thermal distribution of electrons such that:

$$\frac{dW}{dV dt d\omega} = \frac{\int_{v_{min}}^{\infty} P v^2 e^{\frac{-mv^2}{2kT}} dv}{\int_0^{\infty} v^2 e^{\frac{-mv^2}{2kT}} dv}$$

Integrating this expression from

$$v_{min}$$

on to account for a given energy cutoff

$$h\nu$$

and solving yields:

$$\frac{dW}{dV dt d\nu} = 6.8 * 10^{-38} Z^2 n_e n_i T^{\frac{1}{2}} e^{\frac{-h\nu}{kT}} g_{ff}$$

which has units of $ergs * s^{-1} * cm^{-3}$

Finally, the result given for power radiated is proportional to density and temperature by the following relation:

$$P \ \alpha \ n^2 T^{\frac{1}{2}}$$

Relativistic Bremsstrahlung and Bremsstrahlung Applications

Among the two principle applications of bremsstrahlung in astrophysics and astronomy, an important distinction must be made between two different types of bremsstrahlung emission. Emission in the ICM of galaxy clusters is relativistic bremsstrahlung, while that in HII cloud regions is non-relativistic, often simply referred to as thermal bremsstrahlung. Relativistic bremsstrahlung, conditionally defined by bremsstrahlung radiation in which particle velocities are a significant fraction of the speed of light, generally behaves through the same process as that analyzed above, by with an addition factor of γ^2 to account appropriately for relativistic factors.

Relativistic bremsstrahlung abounds as X-rays from high-energy galaxy clusters, such as those made of of NGC X-ray galaxies.These regions, following the logic of the analysis of the previous section, must come from high-density, high-temperature regions. Indeed, the temperature of the ICM in these clusters is found to be on the order of $10^7 - 10^8 keV$, corresponding to particle energies of 0.5 to 5.0+ keV.The emission in these ares follows a power law distribution. In research, sections of this power law can be selectively binned during analysis to isolate emission of a certain region of the spectrum (soft/hard X-rays, etc.) These will correspond to bremsstrahlung caused by certain ions, such as oxygen, silicon, or iron. Binning effectively isolates velocity/energy populations of bremsstrahlung radiation.

Radio-regime bremsstrahlung in H II regions is found in regions of large clouds of ionized Hydrogen, such as the objects M42 and M82. In considering these electron-ion interactions, the only significant factors considered are that of the interactions and radiation from ions. Again, other considerations complicate the analysis beyond a scientifically useful level. These regions must converse be low density and low temperature compared to that of the X-ray ICM. Indeed, the temperature of H II regions is found to be on the order of 10^4 K, and energies to be on the rough order of 1 keV. Energy is given as a function of temperature in these regions by the simple expression of:

$$E = \frac{2kT}{3}$$

It is also worthwhile to consider how bremsstrahlung radiation functions as one of several sources of radiation from astrophysical objects. In both radio and X-ray regimes, synchrotron and thermal radiation make significant contributions to total radiation from a given object. An example using real collected data from M82 is shown in the Figure below. The solid line represents the total radiation spectrum, while the dotted line that is roughly horizontal represents contributions exclusively from bremsstrahlung. We see that bremsstrahlung dominates the spectrum in the region from 30 to 200 GHz. At frequencies higher than this, thermal radiation dominates, while at frequencies lower than this, synchrotron radiation dominates.