Determining Distances: The Baade-Wesselink Method

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1 Introduction and Applications

The Baade-Wesselink method (denoted "BW method" hereafter) was initially developed as a means of testing the pulsation hypothesis for Cepheid variable stars. The pulsation hypothesis—that the period of a Cepheid variable is purely a function of its luminosity—is the underlying assumption of the Cepheid period-luminosity relation, a powerful distance determination method that can be used in galaxies within the Local Group. The BW method bears the name of German astronomer Walter Baade, who postulated in 1926 that the mean radius of a Cepheid variable can be determined absolutely from its brightness, color, and radial velocity, and Adriaan Jan Wesselink, a Dutch astronomer who specialized in variable objects and improved upon Baade's technique twenty years later. Aside from validating the theory that the change in brightness of a Cepheid variable is due to its radial pulsation, the BW method allows for the distance to an individual Cepheid to be determined absolutely.

The BW method can used to determine the absolute luminosity of and therefore the distance to any stellar object for which the photosphere can be approximated as a spherically symmetric shell that is expanding radially outward. The most common targets for distance determination via the BW method are Cepheid variables and other radially pulsating stars such as RR Lyrae variables. However, this method is not limited to stars that pulsate with regular periods; the BW method can also be used during the expansion phase of a supernova, when the outwardly expanding shell is still roughly spherical. With some adaptation, the BW method can be used for stars that do not pulsate or expand radially, but instead undergo non-radial modes of pulsation, such as δ Scuti stars, β Cepheids, and even some types of white dwarfs (Balona & Stobie 1979).

2 Method

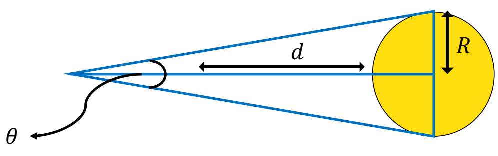

Figure 1: The simple geometry necessary for the BW method.

The geometry used in the BW method is shown in Figure 1. The basic method is very simple: determine the angular radius and the physical radius of a stellar object, which are related simply by the equation

$$\frac{R}{d} = \tan\frac{\theta}{2} \approx \frac{\theta}{2}$$

where R is the physical radius of the star, d is the distance to the star, and θ is the angular diameter of the star. The true measured quantities are the change in the angular diameter and the change in the physical radius, which are similarly related by

$$\frac{\Delta R}{d} = \frac{\Delta \theta}{2}.$$

The necessary quantities are determined through a combination of photometry and spectroscopy.

First, photometry is used to determine the change in the angular radius of the star. By observing the star's pulsation, we obtain a light curve that traces the change in the star's luminosity as it expands and contracts. The measured flux over the pulsation period is converted to apparent magnitude based upon the specifics of the instrumental setup. The apparent magnitude, taken here to be m_v, the apparent visible magnitude, is converted to the apparent bolometric magnitude by applying a phase-dependent bolometric correction as follows:

$$m_{bol}(\phi) = m_v(\phi) + BC(\phi).$$

In order to determine the change in the angular radius, the effective temperature must be determined; this is usually obtained from the photometric color of the star, often reported in the V-K magnitude (e.g. Gieren et al. 2012). Finally, by using

$$m_{bol}(\phi_2) - m_{bol}(\phi_1) = -10 \log \frac{T_{eff}(\phi_2)}{T_{eff}(\phi_1)} - 5 \log \frac{R_p(\phi_2)}{R_p(\phi_1)},$$

the ratio of the photospheric radii from two points in the star's phase curve, $R_p(\phi_2)/R_p(\phi_1)$, can be determined in arbitrary units (Böhm-Vitense et al. 1989).

The change in the object's physical radius is determined spectroscopically. The radial velocity curve of the stellar photosphere during the star's pulsation is obtained from the Doppler shift of spectral lines. The displacement (the change in the radius) is determined by integrating the radial velocity curve over the entire pulsation period:

$$\Delta R = \int_{\phi_1}^{\phi_2} v \, dt.$$

2.1 Geometric BW Method

Although the first iteration of the BW method used photometric methods to determine the change in the star's angular radius, advances in interferometry have given rise to the geometric BW method. The two observation techniques used for the geometric version of the BW method are high-resolution spectroscopy and interferometry. The former provides the radial velocity curve over the pulsation cycle of the star which, when integrated, provides the linear radius variation of the star in physical units. The interferometric observations directly document variation of the star's angular radius. The ratio of these two quantities gives the distance of the Cepheid.

2.2 Application to Supernova Expansion

The BW method can be applied to supernovae shortly (in astronomical terms) after expansion begins, while the expanding shell is still roughly spherical and an appreciable amount of mass has not yet been swept up, slowing the shell's expansion. The photometric angular radius is given by $\theta_{ph}^2 \sim f_\nu / F_\nu$, where f_ν is the observed flux from the supernova and F_ν

is the theoretical absolute flux at a given frequency. The expansion velocity of the shell is determined spectroscopically by the usual method. The distance can then be determined from the relation between the angular and physical radii:

$$\theta_{ph} \sim (R_{SN} + v\Delta t)/d.$$

Here, R_{SN} is the radius of the supernova progenitor and Δt is the time since the supernova expansion began (Isern et al. 1989).

3 Problems and Limitations

3.1 The Blackbody Assumption

The classical BW method assumes that the target object radiates as a perfect blackbody, an assumption that we know to be false due to the presence of hundreds of absorption lines in stellar spectra. This fallacious assumption doomed the first attempt to use the BW method in 1928; the radial velocity and surface brightness curves were found to be out of phase and no distance could be determined (Wesselink 1946). Later, the assumption of a perfect blackbody was replaced with the assumption that there is a single-valued relation between the surface brightness and temperature of an object (the perfect blackbody assumption is a special case of this assumption). Today, we largely maintain this assumption, but continue to refine the technique; one outstanding issue is the disparity between distances derived from different photometric methods due to the fact that different temperatures are inferred from different filter combinations (Böhm-Vitense et al. 1989).

3.2 Corrections to the Radial Velocity

The radial velocity measured spectroscopically is not necessarily equal to the expansion velocity. This is primarily due to the fact that spectral lines can arise from layers that do not necessarily move at the pulsation velocity. In addition, hydrodynamic flows within the outer layers of the star can affect the observed radial velocity. It is also important to consider the optical depth of the relevant layers probed in this method; the angular and physical radii are determined from observations of two different optical depths. The angular radius is determined via measurements of the star's periodically varying surface brightness, which is measured from the base of the photosphere, which for an unresolved source is at an optical depth of 2/3. However, spectral lines, the basis of the physical radius measurement, are formed at much lower optical depths. Because of this discrepancy, corrections are necessary to account for the fact that these two methods probe different regions of the star.

In addition, because stars are spheres and not disks, the observed radial velocity is only equal to the pulsation velocity at the center of the star. However, spectral lines do not arise only from the center of the star's disk, and it is necessary to define a projection factor, p, equal to V_{puls}/V_{rad} to account for this. The pulsation and radial velocities are illustrated in Figure 2. The projection factor is not only a geometric effect, but also depends upon the extent of limb darkening.

Because stars have steep temperature gradients from the extreme temperatures necessary to maintain fusion in stellar cores to the observed effective temperatures of a few thousand to a few tens of thousands of degrees at the surface, the limb of the star appears darker than the center. Limb darkening also affects the determination of the projection factor, p. For example, the projection factor for a uniform disk is equal to 3/2. For the case where the limb is completely dark, the projection factor falls to 4/3. In general, the projection factor ranges from 1 to 3/2 and varies from star to star (Marengo et al. 2004).

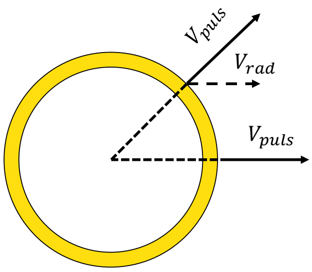

Figure 2: Illustration of the discrepancy between the pulsation velocity (the desired quantity) and the radial velocity (the measured quantity).

4 Summary

The BW method is a primary distance indicator initially developed as a test of the pulsation theory of Cepheid variable stars. It relies on simultaneous determination of the angular and physical radius of a stellar object via spectroscopy and either photometry or interferometry. The BW method is used today to determine absolute luminosities of and distances to Cepheid variables and other radially pulsating variable stars, supernovae, and certain types of non-radially pulsating stars. Although the BW method has several issues, it is a powerful tool, and the distance to individual Cepheids is an important baseline for the Cepheid period-luminosity relationship.

References

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